

# The “S” Curve Relationship between Export Diversity and Economic Size of Countries

Lunchao Hu, Kailan Tian, Xin Wang and Jiang Zhang

*Department of Systems Science, School of Management, Beijing Normal University,  
Beijing, China, 100875*

---

## Abstract

The highly detailed international trade data among all countries in the world during 1971-2000 shows that the kinds of export goods and the logarithmic GDP (gross domestic production) of a country has an S-shaped relationship. This indicates all countries can be divided into three stages accordingly. First, the poor countries always export very few kinds of products as we expect. Second, once the economic size (GDP) of a country is beyond a threshold, its export diversity may increase dramatically. However, this is not the case for rich countries because a ceiling on the export diversity is observed when their GDPs are higher than another threshold. This pattern is very stable for different years although the concrete parameters of the fitting sigmoid functions may change with time. In addition, we also discussed other relationships such as import diversity with respect to logarithmic GDP, diversity of exporters with respect to the number of export goods etc., all of these relationships show S-shaped or power law patterns. Although this paper does not explain the origin of the S-shaped curve, it may provide a basic empirical fact and insights for economic diversity.

*Keywords:* Economic Diversity, Logistic Curve, International Trade

---

## 1. Introduction

Diversity phenomenon exist widely in economic systems. As we observed, individual consumptions become more diversified when people get

---

*Email address:* zhangjiang@bnu.edu.cn (Lunchao Hu, Kailan Tian, Xin Wang and Jiang Zhang)

richer[1]; organizations demand for more diverse human resource, products and investments when they become larger[2]; and countries can export more kinds of products and services to other countries as their economies are stronger. Nathan et.al. has discussed the relationship between the network diversity and economic development by a large data set of individual communication[3]. Templet discovered that the economic diversity generally increases in regional economy[4]. However, so far, this kind of researches who addressed the diversity phenomena in economics, especially the quantitative patterns based on empirical data, are very sparse.

Nevertheless, compared to the situation in economics, there are many studies on diversity phenomena in ecology both from the empirical data and theoretical models[5–8]. We can borrow some useful conceptions and methods from the ecology to study the economic diversity because these two “eco” systems have lots of similarities and commonalities[9, 10]. For example, the relationship between the number of species and the area[5], or the productivity[8] of the habitat have been widely discussed in ecology. This relationship inspired us to find the similar relationship between the export diversity which is compared to the species diversity and the economic size (GDP) of a country which is compared to the area or productivity of a habitat in international trade.

Although Krugman et al. have touched the problem of the relationship between the number of different kinds of export goods (export diversity) and the size of a country (population) in their classic international trade model[11, 12], they simply assumed that this relationship is linear without any support of empirical data[13, 14]. Recently, Petersson has studied the export diversity and intra-industry trade in Southern Africa[15]. Johansson and Nilsson reported R&D accessibility and regional export diversity in Sweden[16]. Although these studies were based on the empirical data, they only focused on a specific country or region but not the universal pattern on the global.

In this paper, we use the highly detailed international trade data during 1971-2000 to investigate the relationship between the export diversity of a country which is measured by counting the categories of export goods and the size of an economy which is measured by its GDP. Based on the data, we find that a nation with higher GDP level will indeed export a wider variety of goods, however, this dependence can be described by an S-shaped curve (fitted by a logistic equation) which is one of the most important universal patterns in complex systems[17–19] rather than the common linear or power

law relations[5]. Furthermore, we find that the S-shaped curve is ubiquitous because it can be applied to the relationships between other diversity-related variables such as the number of different kinds of import goods, the number of exporters, etc. and economic size. While the relationships between any pair of the diversity-related variables can be described by power laws.

The outline of the paper is as follows. In section 2, we briefly introduce the data source. Section 3 proposes our main results. Subsection 3.2 discusses the classification of countries into three stages. We show how robust and stable the pattern is in subsection 3.3. And also, we discuss how the key parameters in S-shaped curves indicate the dynamical change of the global economy. In section 4, more relationships on diversity of international economics are discussed. At last, section 5 points out that although this paper does not provide any explanation of the S-shaped curves, it reveals a robust and promising pattern which is the first step of the study on the economic diversity phenomenon.

## 2. Data

Our analysis is based on the world GDP statistics from the World Bank web-site ([www.worldbank.org](http://www.worldbank.org)) and the bilateral trade statistics from NBER-UN world trade database ([www.nber.org/data](http://www.nber.org/data)). The former records the name of nearly 240 countries, as well as their GDP, population, etc. from 1971 to 2006, while the latter records the importer, exporter, 4-digit categories, the value and quantity of each bilateral trade from 1962 to 2000[20].

When dealing with the NBER-UN trade data, we calculated the total number of the categories of export commodities[5] for each country as the measurement of the export diversity. We have also tried other indices such as Shannon entropy to measure the export diversity[15, 16], but they can't give universal and well-fitted patterns. The countries without the export diversity or GDP data are ignored. Thus, we got the full data of 126 countries from 1971 to 1991, 148 countries from 1992 to 2000.

The NBER-UN trade data were constructed from United Nations trade flows over two periods: (i) 1962-1983, the data for which covered all trade flows that different products were classified by SITC Rev.1 standard (Standard International Trade Classification, Revision 1) and trading partners with country codes; (ii) 1984-2000, same as the first data set except the products were classified by SITC Rev.2 standard.

It is necessary to mention some historical background of the Standard International Trade Classification(SITC). SITC, Revision 1, represented an improvement over the original SITC due to the increase of the volume of trade and the diversity of commodity during 1960-1968 as well as the limitation of the SITC. And also, in 1984, the Economic and Social Council further replaced the SITC Rev.1 by SITC Rev.2 because of the same reason. Therefore, we know that it was the expansion of commodity diversity itself caused the alteration of the classification standard but not the opposite. Hence, our results are not dependent on the change of the classification standard.

### 3. Results

#### 3.1. S-shaped curve

As the gravity model of trade theory has found, the trade flows (export/import volume or value) between any countries can be described by a Newtonian gravity-like equation[21, 22]. That means the trade flows are the power law functions[23] of the economic size of the importer or exporter. However, the export diversity has a very different dependence on GDP. Based on the collected data, we plotted the original data and the estimated curve in Fig 1.

From the data shown in Fig 1, we know that an S-shaped curve may fit them very well. We use the logistic function like the following Equation:

$$y = \frac{A}{1 + e^{-k(x-X_M)}}, \quad (1)$$

which are widely used in many disciplines[17] to characterize the empirical S-shaped data. In equation 1,  $y$  is the export diversity measured by the number of different categories of the export goods, and  $x$  represents  $\log(\text{GDP})$ .  $A$ ,  $X_M$  and  $k$  are parameters to be estimated with the given data.  $A$  is the upper limit of the diversity of commodities.  $k$  is the slope on the inflection point and represents the maximum relative growth rate of diversity.  $X_M$  stands for the logarithmic GDP size at the inflection point (see Fig 1).

The S-shaped curve can be divided as three stages: in the first stage, when a nation's economic size (GDP) stays in a low level, the number of export goods is naturally small and increases with a slower speed than the growth of GDP; in the second stage, their GDPs climb up to a relatively high level, and the growth rate of the diversity increases dramatically; in the

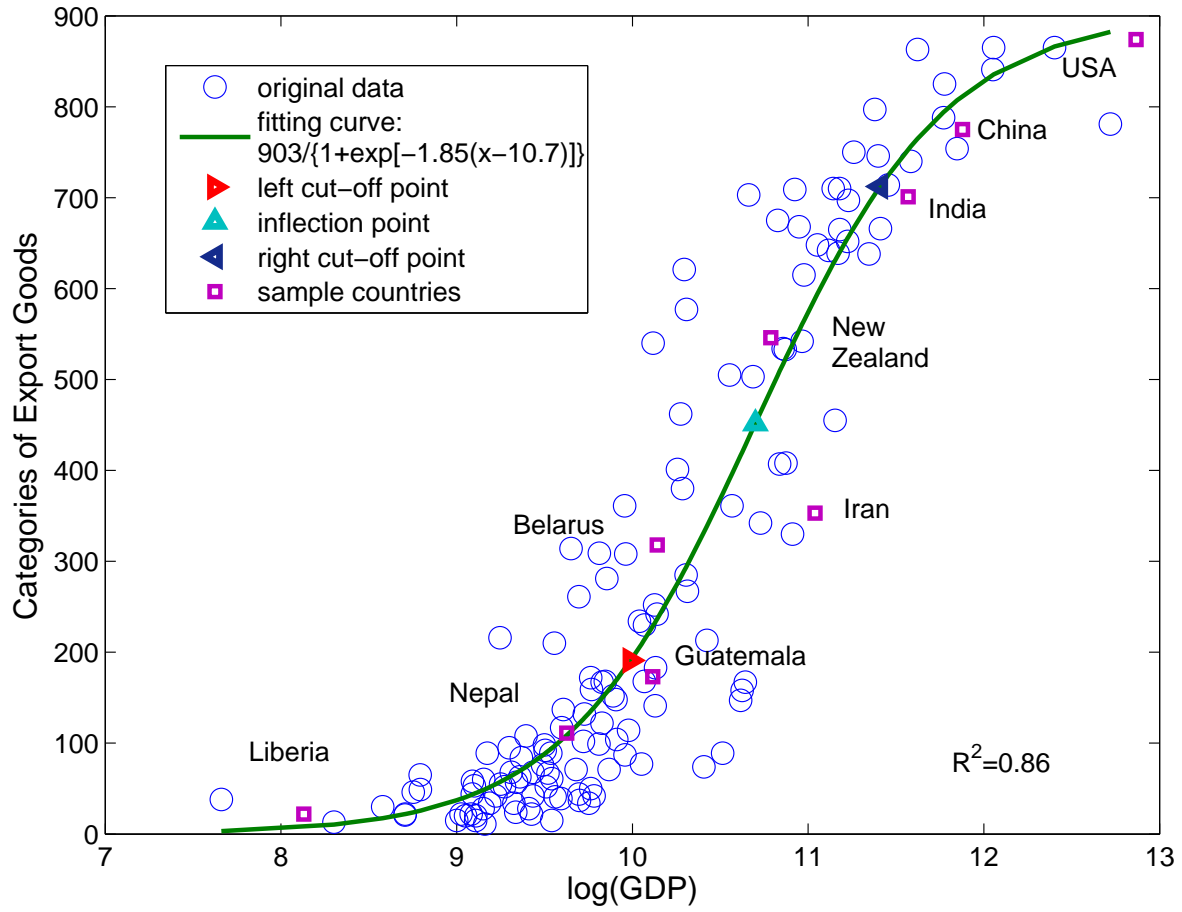


Figure 1: The original data and the “S” curve fitting of export diversity and log(GDP) for all countries in 1995. The curve has three stages: initial stage, acceleration stage and final stage, which are determined by the three critical points (left cut-off point, inflection point, and right cut-off point) which are denoted as triangles. Besides, some countries are picked out and are marked by the squares.

last stage, the growth rate of diversity slows down and gradually falls behind GDPs again.

By means of Least Square (LS) method, we obtained the estimated values of parameters and some essential statistical data (R-square=0.86 and F-value=231). Accordingly, we claim that the logistic model can fit our data very well. Besides, we also have studied the residuals of each empirical data and found that the residuals are larger for the middle-class countries, say in the second stage. Therefore, the export diversity can be predicted by the logistic function better in the first and third stages.

### 3.2. Three stages

From Fig 1, we know that the curve can be divided into three stages, namely initial stage, acceleration stage, and final stage. However, there is no any unified and rigorous method to identify them. In this paper, we use the three order derivative of the logistic function (Equation 1) to determine the stages accordingly,

$$d^3\left(\frac{A}{1 + e^{-k(x-X_M)}}\right)/dx^3 = 0. \quad (2)$$

Solving this equation, we can obtain two non-zero solutions, namely the left cut-off point  $(x_L, y_L)$  and the right cut-off point  $(x_R, y_R)$ . We know that the left cut-off point which can separate the initial stage and accelerating stage should maximize the curvature of the curve. Therefore, setting the third order derivative equaling zero can obtain this separating point[24]. The same method follows for the right cut-off point separating the accelerating stage and the final stage.

There is another important point called inflection point  $(X_M, Y_M)$  whose curvature is zero indicating the growth rate of the diversity reaches the maximum. We can get the analytic expressions of these important points in Table 1.

Until now, the estimated curve in Fig 1 can be divided into three stages by the two cut-off points. In the initial stage, there are 71 countries including the poorest countries and some developing countries such as Liberia, Nepal etc. Most of them locate in Africa and Asia with less developed industries and shortage of natural resources. The acceleration stage with 56 countries contains some developing and developed countries. Iran, the second largest oil exporter, and Belarus, with developed industry and advanced agriculture, are all developing countries and locate below the inflection point. While, the developed countries, such as New Zealand, Norway and Sweden, are at a

Table 1: The Coordinate of Three Important Points  
(We classify all the sample countries into three stages according to them.)

	Left Cut-off Point ( $X_L, Y_L$ )	Inflection Point ( $X_M, Y_M$ )	Right Cut-off Point ( $X_R, Y_R$ )
X-coordinate	$X_M - \frac{\ln(2+\sqrt{3})}{k}$	$X_M$	$X_M - \frac{\ln(2-\sqrt{3})}{k}$
Y-coordinate	$\frac{A}{3+\sqrt{3}}$	$\frac{A}{2}$	$\frac{A}{3-\sqrt{3}}$

high speed of diversity growing with a falling acceleration above the inflection point. Finally, the third stage contains the most developed countries such as USA and Japan, with broad geographic and well-developed industries. However, China, with GDP topped the ninth in 1995, is also included in this stage. Guatemala and India both are around the cut-off points. The former one will welcome to a period of accelerating growth of export diversity while the later one has experienced the glorious moments and will step into a period of lower diversity growth rate.

### 3.3. Dynamics

So far, we have studied the S-shaped pattern between export diversity and  $\log(\text{GDP})$  only in one year. To look deeper into this pattern especially on time, we fitted the collected data from 1971 to 2000 in the same way. However, because of the change of classification method in 1983 mentioned in section 2, we normalized the diversity data with the method below,

$$Y'_i = \frac{Y_i - \min \{Y_i\}}{\max \{Y_i\} - \min \{Y_i\}}, \quad (3)$$

where  $Y_i$  is the number of export goods categories of country  $i$ ,  $Y'_i$  represents the normalized diversity which ranges from 0 to 1. Fig 2 shows the patterns in four different years with normalized data.

In Fig 2, two cut-off points and the inflection point are plotted in each year. We can see that the S-shaped pattern is stable in these thirty years. The shift and the expansion of the curves imply that the S-shaped pattern is more obvious after 1990s. This phenomenon is also reflected by the goodness of fitting in the inset of Fig 2.  $R^2$  increases from 0.75 to around 0.83 in 1983, and fluctuates a little in the following years.

We can trace one specific country, China, which was undergoing dramatic economic development in these years. We know that China has adopted her

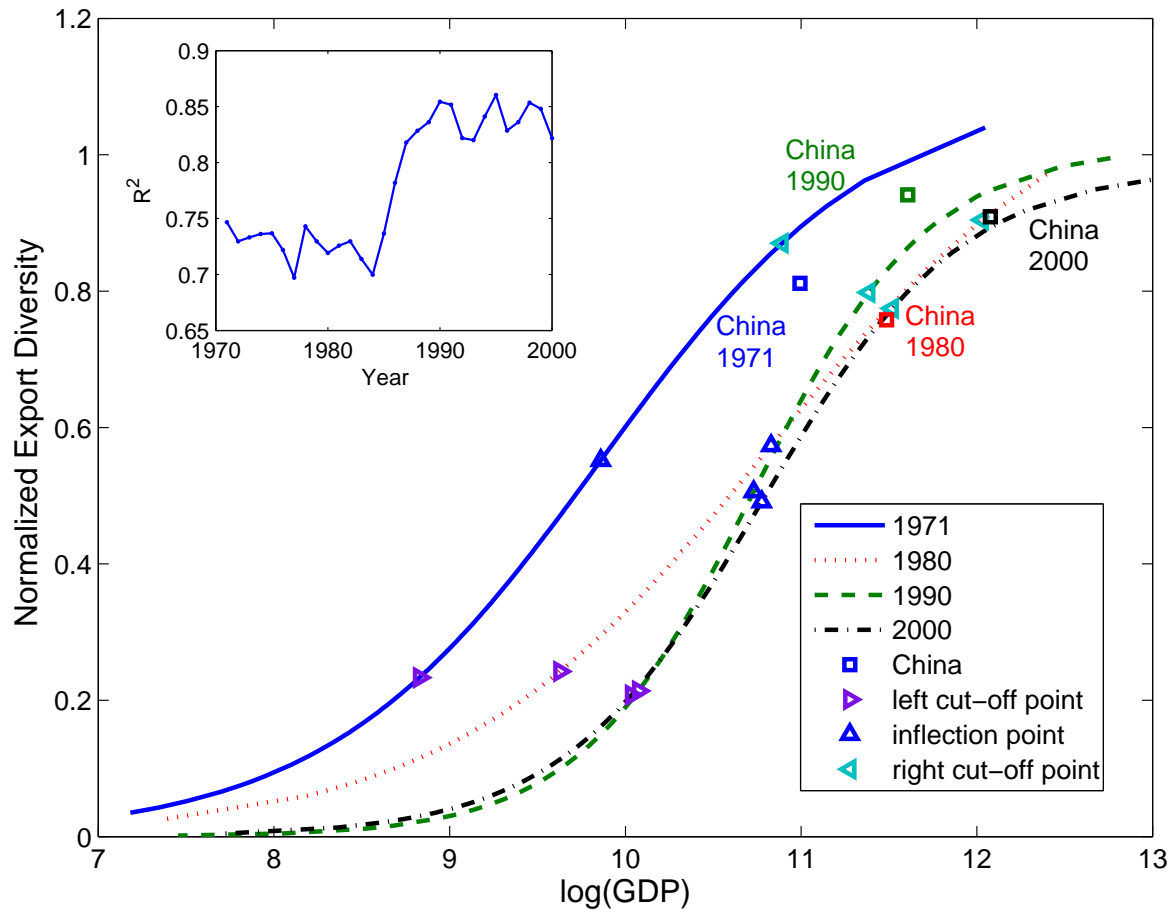


Figure 2: The relationship between normalized diversity of export goods and  $\log(\text{GDP})$  in four selected years. And the goodness of fitting (R-square) in thirty years is shown in the inset.



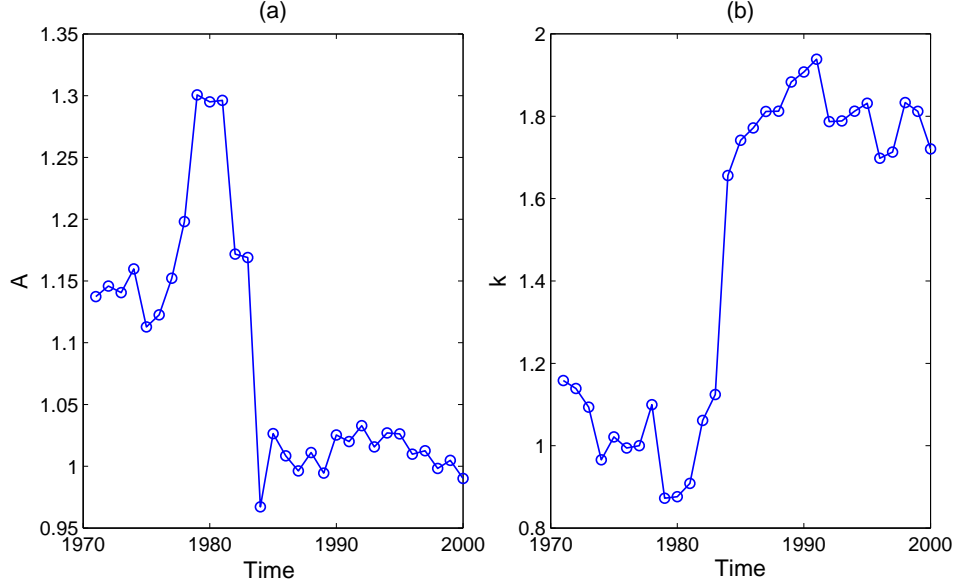


Figure 3: The fluctuations of (a). $A$  (the upper asymptotes) and (b). $k$  (the slope of the inflection point) in thirty years

opening policy from 1978, so she stayed in the acceleration stage in 1971 and 1980 in Fig 2, meaning that the growth rate of the diversity increased rapidly. After that, as her GDP ranked higher, the growth rate of the diversity slowed down. Thus, China stepped into the final stage.

From Fig 3(a) we know that  $A$  fluctuates more intensive before 1983 and then waves a little around 1. That means the S-shaped relation is more stable after 1983. Fig 3(b) shows the change of  $k$  which is the relative maximum growth rate of export diversity in the thirty years. We know the value of  $k$  keeps increasing from 1.13(1981) to 1.81(1990) while remains relatively stable from 1971 to 1979 and 1991 to 2000. Thus, the S-shaped curves became steeper suddenly in about 1980-1985. This phenomenon can be also observed from the dynamics of the three critical points. From Fig 4 we know the change of  $X_L$ ,  $X_M$  and  $X_R$  values follow similar paths. In the first decade, the three variables increase continually, and they remain 10.0, 10.7 and 11.5 respectively with only small fluctuations after 1983. However,  $Y_L$ ,  $Y_M$  and  $Y_R$  values do not have much change and remain around their average value. Hence, the S-shaped curves move right gradually at first and stop

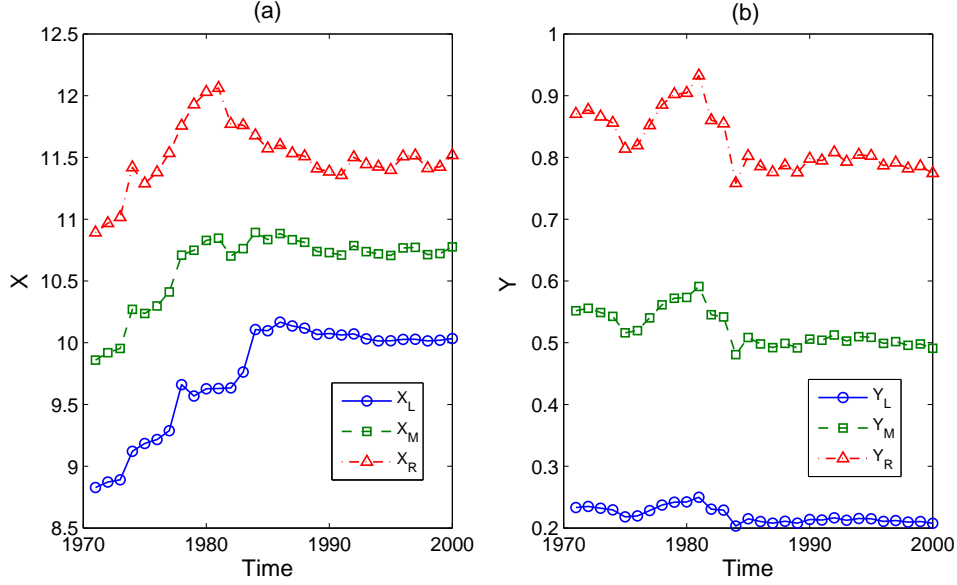


Figure 4: The dynamical trends of the two cut-off points and the inflection point during the 30 years (a). X-coordinates, (b).Y-coordinates.

after 1980's. That indicates the global economy kept increasing during the first decade and then slowed down.

We believe that the dynamics of these parameters, especially the big fluctuations around 1983, indicate the changed situation of the international trade environment[21]. First, we know that there was a serious global oil crisis in 1970s, as a result, the system of international trade changed in 1980s all around the world. Many countries abandoned the traditional inward-looking trade, turned the export-substitution model of economic growth to export-oriented model. Second, the adjustment of the basic goal of market-oriented economic structure had been made because of the boom of information and service industries in 1980s. Furthermore, in order to adapt for the changes of the trade environment, many countries made some change in 1980s. For instance, the United State of America changed their trade policy to multilateral free trade system due to the oil crisis and depreciation. Japan, as another example, also strengthened the multilateral trading system and developed the global economy since 1980s.

Thus, when the environment of the international trade turns better, all

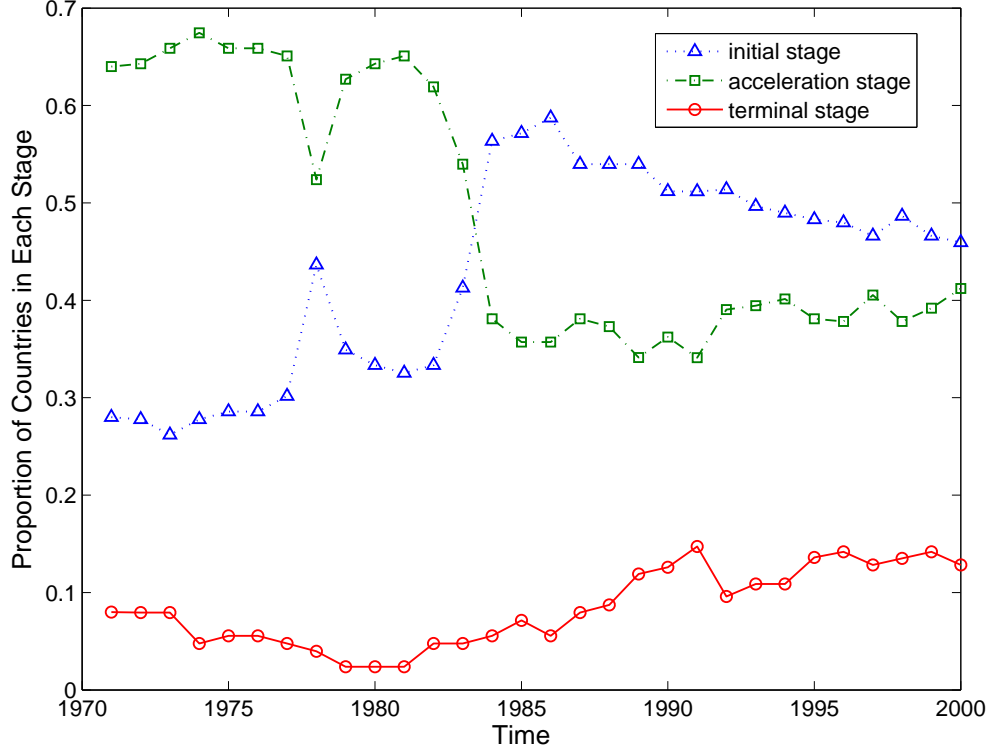


Figure 5: The proportions of countries in each stage change with time during the 30 years

countries tend to export more goods. This can be reflected by the maximum growth rate of export diversity,  $k$ , which can indicate the international trading circumstance. The environment changes better as it jumps to a higher level.

Because  $k$  increased and the total number of countries in the world almost unchanged in these years, we know that the proportion of the countries in each stage must have changed. From Fig 5 we know that the proportion of the countries in the initial stage increased while those in the acceleration stage fell down during 1980 to 1985. After 1985, both fluctuated little and the difference between them became smaller. However, the proportion in the final stage almost remained around 0.1 all the time. This phenomenon may be accounted for the unbalanced development in different countries in the reform

Table 2: All the S-shaped Relationships

$x$	$y$	$A$	$k$	$X_M$	$R^2$
log(GDP)	Importers	203	1.09	10.94	0.89
log(GDP)	Exporters	197	1.42	10.75	0.92
log(GDP)	Import Goods	749	1.66	9.65	0.87
log(GDP)	Export Goods	903	1.85	10.7	0.86

of international trade policies as mentioned before. As the globalization process started, the gap of the export diversity between the rich and poor countries became larger. Consequently, the countries which had climbed to the acceleration stage fell down into the initial stage again.

#### 4. Discussion

In previous sections, we have discussed the S-shaped curves of the relationship between export diversity and log (GDP). However, we have also the data of the other variables including the number of importers, the categories of import goods, and the number of exporters. So, we can explore more empirical patterns of diversity. In this section, we will point out that all the bivariate relationships fall into two classes, namely, S-shaped relationships and power law relationships.

##### 4.1. S-shaped relationships

We find that other diversity-related variables including the categories of import goods, the number of distinct exporters and the number of distinct importers are all have S-shaped relationship with the logarithmic GDP for each country. Fig 6(a) shows the relationship between the number of exporters and log(GDP) as an example, and all these “S” curves are concluded in Table 2.

As shown in Table 2, we know that the logistic function can fit all the relationships between log (GDP) and importers, exporters, import goods and export goods with high significances ( $R^2$ ). Nevertheless, the parameters ( $A, k$  and  $X_M$ ) are very different which can also be divided into two groups. We know that  $A$  is the upper limit and represents the range of diversity of trade. From Table 2, we observe that  $A_{import} < A_{export}$  for importers/exporters but  $A_{import} > A_{export}$  for import/export goods. That means countries are more likely to export fewer kinds of goods to more diverse countries. And

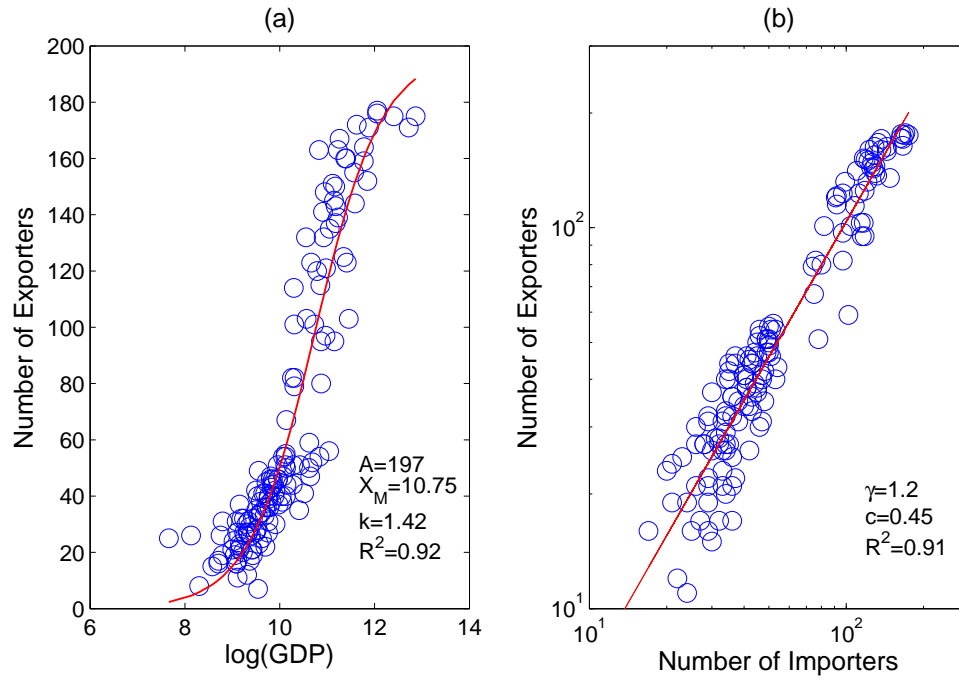


Figure 6: Other relationships about diversity in 1995. (a). The S-shaped curve of the relationship between the number of exporters and the logarithm of GDP. (b). The power law relationship between the number of importers and exporters.

Table 3: All the Power Law Relationships

$X$	$Y$	$c$	$\gamma$	$R^2$
Importers	Exporters	0.45	1.2	0.91
Importers	Export Goods	0.15	1.75	0.78
Import Goods	Exporters	0.052	1.14	0.71
Import Goods	Export Goods	0.0015	1.92	0.79
Importers	Import Goods	21.32	0.74	0.66
Exporters	Export Goods	0.51	1.46	0.84

$k$  is the maximum specific growth of  $y$ . So  $k_{import} < k_{export}$  always holds for countries and goods, which means the export diversity grows faster than import diversity as GDP grows.

#### 4.2. Power-law relationships

Scaling property is one of the most important universal quantitative laws governing different dynamics of complex systems [9, 23, 25–29]. In paper[23], the authors have shown that the import value and export value of a country all have power law relationships with respect to GDP. In this paper, we find the variables relating to diversity also follow the power law relationship:

$$Y = cX^\gamma, \quad (4)$$

where,  $X$  and  $Y$  are diversity-related variables such as the number of importers, categories of export goods etc.  $c$  and  $\gamma$  are parameters to be estimated. By using equation 4 to fit our empirical data, we obtain Fig 6(b) and Table 3.

Fig 6(b) shows the relationship between the number of importers and exporters as an example. From Table 3, we know that all power law relationships are significant because their  $R^2$ s are all larger than 0.6. And all the relationships between import properties and export properties are super-linear[29] for  $\gamma > 1$ , which means that with the increase of import properties, export properties can grow faster. So do the relationship between exporters and categories of export goods, that is to say one country may have more exporters for each export goods as the export goods diversity increases. However, the relationship between importers and import goods is sub-linear[29] meaning that the growth speed of the categories of import goods is slower than the number of importers.

## 5. Concluding Remarks

We have shown the S-shaped relationship between the number of different kinds of export commodities and log GDP, which is well fitted by the logistic function. We have also found the two cut-off points which can divide the curve into three stages. In addition, we analyzed the dynamics of the parameters. Particularly, the parameter  $k$  increases suddenly in around 1980's which is supposed to indicate the change of international trade environment. Finally, we also discussed the other possible diversity-related properties all have S-shaped relationships with respect to log GDP and power law relations with other variables.

Although this paper did not give any explanations about the origins of S-shaped curves and power law relationships, we believe that we have made a first step to understand the economic diversity phenomenon because we have revealed robust and universal patterns based on the empirical data.

*Acknowledgement.* We acknowledge the web site [www.nber.org/data](http://www.nber.org/data) to provide free data about international trade. This research was supported by National Natural Science Foundation of China under Grants of No. 61004107

## References

## References

- [1] C. Anderson, The Long Tail: Why the Future of Business is Selling Less of More, Hyperion, 2006.
- [2] S. E. Page, The Difference: How the Power of Diversity Creates Better Groups, Firms, Schools, and Societies, illustrated Edition, Princeton University Press, 2007.
- [3] N. Eagle, M. Macy, R. Claxton, Network diversity and economic development, Science 328 (5981) (2010) 1029–1031. doi:10.1126/science.1186605.
- [4] P. H. Templet, Energy, diversity and development in economic systems; an empirical analysis, Ecological Economics 30 (2) (1999) 223–233. doi:10.1016/S0921-8009(98)00085-8.
- [5] S. P. Hubbell, The Unified Neutral Theory of Biodiversity and Biogeography, Princeton University Press, 2001.

- [6] K. Gaston, J. Spicer, *Biodiversity: An Introduction*, 1st Edition, Wiley-Blackwell, 1998.
- [7] C. P. H. Mulder, D. D. Uliassi, D. F. Doak, Physical stress and diversity-productivity relationships: The role of positive interactions, *Proceedings of the National Academy of Sciences of the United States of America* 98 (12) (2001) 6704–6708. doi:10.1073/pnas.111055298.
- [8] M. Loreau, S. Naeem, P. Inchausti, J. Bengtsson, J. P. Grime, A. Hector, D. U. Hooper, M. A. Huston, D. Raffaelli, B. Schmid, D. Tilman, D. A. Wardle, Biodiversity and ecosystem functioning: Current knowledge and future challenges, *Science* 294 (2001) 804–808.
- [9] J. Chave, S. Levin, Scale and scaling in ecological and economic systems, *Environmental & Resource Economics* 26 (4) (2003) 527–557.
- [10] J. C. Nekola, J. H. Brown, The wealth of species: ecological communities, complex systems and the legacy of Frank Preston, *Ecology Letters* 10 (2007) 188–196.
- [11] P. R. Krugman, Increasing returns, monopolistic competition, and international trade, *Journal of International Economics* 9 (4) (1979) 469–479. doi:10.1016/0022-1996(79)90017-5.
- [12] E. Helpman, P. Krugman, *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy*, The MIT Press, 1987.
- [13] P. S. Armington, A theory of demand for products distinguished by place of production, *Staff Papers - International Monetary Fund* 16 (1) (1969) 159–178. doi:10.2307/3866403.
- [14] P. R. Krugman, Intraindustry specialization and the gains from trade, *The Journal of Political Economy* 89 (5) (1981) 959–973.
- [15] L. Petersson, Export diversification and intra-industry trade in south africa, *South African Journal of Economics* 73 (4) (2005) 785–802.
- [16] S. Johansson, C. Karlsson, R&D accessibility and regional export diversity, *Annals of Regional Science* 41 (3) (2007) 501–523.



- [17] M. H. Zwietering, I. Jongenburger, F. M. Rombouts, K. Riet, Modeling of the bacterial growth curve, *Applied and Environmental Microbiology* 56 (6) (1990) 1875–1881.
- [18] L. Haanstra, P. Doelman, J. H. O. Voshaar, The use of sigmoidal dose response curves in soil ecotoxicological research, *Plant and Soil* 84 (2) (1985) 293–297. doi:10.1007/BF02143194.
- [19] R. Northam, *Urban Geography*, John Wiley & Sons, New York, 1979.
- [20] C. A. Hidalgo, B. Klinger, A. Barabási, R. Hausmann, The product space conditions the development of nations, *Science* 317 (5837) (2007) 482 – 487. doi:10.1126/science.1144581.
- [21] P. R. Krugman, M. Obstfeld, *International Economics: Theory and Policy*, 6th Edition, Addison Wesley, 2002.
- [22] J. E. Anderson, The gravity model, *Annual Review of Economics* 3.
- [23] J. Zhang, T. Yu, Allometric scaling of countries, *Physica A: Statistical Mechanics and its Applications* 389 (21) (2010) 4887–4896. doi:10.1016/j.physa.2010.06.059.
- [24] J. WANG, Z. WU, Delimiting the stages of urbanization growth process: A method based on northam’s theory and logistic growth model (in chinese), *acta geographica sinica* 64 (2).
- [25] W. A. Brock, Scaling in economics: A reader’s guide, *Industrial and Corporate Change* 8 (1999) 409–446.
- [26] J. Brown, G. West, *Scaling in Biology*, Oxford University Press, 2000.
- [27] Y. Lee, An allometric analysis of the U.S. urban system: 1960-80, *Environment and Planning A* 21 (1989) 463–476.
- [28] A. Isalgue, H. Coch, R. Serra, Scaling laws and the modern city, *Physica A* 382 (2007) 643–649. doi:10.1016/j.physa.2007.04.019.
- [29] L. M. Bettencourt, J. Lobo, D. Helbing, C. Kuhnert, G. B. West, Growth, innovation, scaling, and the pace of life in cities, *Proceedings of the National Academy of Sciences of the United States of America* 104 (2007) 7301–7306. doi:10.1073/pnas.0610172104.